INFOMAT

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Norsk Matematisk Forening

September 2004

Septembernummeret fylles i stor grad av et utdrag av et interview Raussen og Skau har gjort med Atiyah og Singer. Vi har også med utdrag fra den norske versjonen av rapporten Nils Voje Johansen og Yngvar Reichelt har skrevet for EMS Newsletter.

Om du har stoff som du mener passer for INFOMAT, send et brev til

`infomat at math.ntnu.no`

Hjemmeside: `http://www.matematikkforeningen.no/INFOMAT`

Nytt fra instituttene

Innholdet baserer seg på innsendt informasjon fra enkeltmedlemmer og fra instituttene.

Institutt for matematiske fag (IMF), NTNU

Doktorgrader: *Runar Holdahl* forsvarte sin avhandling for dr.ing.-graden 2. september, tittel *Wavelets and Partial Differential Equations*. Veileder Helge Holden.

Gjester:

Nicole Snashall, University of Leicester, 21. – 28. september
Edward L. Green, VirginiaTech, 1. – 31. september
Roberto Martínez-Villa, Universidad Nacional Autonoma de Mexico, 3. – 17. oktober.

Kjempesatsing på nano-lab ved NTNU I høst starter byggingen av et nytt laboratorium for nanoteknologi ved NTNU. Prosjektet anslås å koste nærmere 220 millioner kroner. Du kan lese mer i Universitetsavisa,

`http://www.universitetsavisa.no/`

Matematisk institutt, UiO

Gjester: *Birgit Richter* (Universität Bonn) er Suprema gjesteforsker ved UiO fra oktober 2004 til januar 2005.

Matematisk institutt, Universitetet i Bergen

Overgang til emeritus-stillinger. Tre av instituttets ansatte har i sommer oppnådd emeritus- status:

Gerhard Berge
Leif E.Engevik
Oddvar Iden

Martin Raussen, Aalborg University, Denmark

Christian Skau, NTNU, Trondheim

Interview with Michael Atiyah and Isadore Singer

Sammen med EMS Newsletter trykker INFOMAT intervjuet med Michael Atiyah and Isadore Singer som Martin Raussen og Christian Skau gjorde i Oslo 24. mai 2004. Les hele intervjuet i EMS Newsletter eller på

`http://www.matematikkforeningen.no/INFOMAT/04/AS.pdf`

The Index Theorem

First, we congratulate both of you for having been awarded the Abel Prize 2004. This prize has been given to you for "the discovery and the proof of the Index Theorem con-

necting geometry and analysis in a surprising way and your outstanding role in building new bridges between mathematics and theoretical physics". Both of you have an impressive list of fine achievements in mathematics. Is the Index Theorem your most important result and the result you are most pleased with in your entire careers?

ATIYAH First, I would like to say that I prefer to call it a theory, not a theorem. Actually, we have worked on it for 25 years and if I include all the related topics, I have probably spent 30 years of my life working on the area. So it is rather obvious that it is the best thing I have done.

SINGER I too, feel that the index theorem was but the beginning of a high point that has lasted to this very day. It's as if we climbed a mountain and found a plateau we've been on ever since.

We would like you to give us some comments on the history of the discovery of the Index Theorem.[1] *Were there precursors, conjectures in this direction already before you started? Were there only mathematical motivations or also physical ones?*

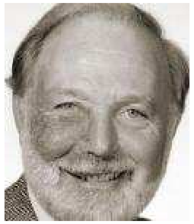
ATIYAH Mathematics is always a continuum, linked to its history, the past - nothing comes out of zero. And certainly the Index Theorem is simply a continuation of work that, I would like to say, began with Abel. So of course there are precursors. A theorem is never arrived at in the way that logical thought would lead you to believe or that posterity thinks. It is usually much more accidental, some chance discovery in answer to some kind of question. Eventually you can rationalize it and say that this is how it fits. Discoveries never happen as neatly as that. You can rewrite history and make it look much more logical, but actually it happens quite differently.

SINGER At the time we proved the Index Theorem we saw how important it was in mathematics, but we had no inkling that it would have such an effect on physics some years down the road. That came as a complete surprise to us. Perhaps it should not have been a surprise because it used a lot of geometry and also quantum mechanics in a way, à la Dirac. [...]

Collaboration

Both of you contributed to the index theorem with different expertise and visions – and other people had a share as well, I suppose. Could you describe this collaboration and the establishment of the result a little closer?

SINGER Well, I came with a background in analysis and differential geometry, and Sir Michael's expertise was in algebraic geometry and topology. For the purposes of the Index Theorem, our areas of expertise fit together hand in glove. Moreover, in a way, our personalities fit together, in that "anything goes": Make a suggestion - and whatever it was, we would just put it on the blackboard and work with it; we would both enthusiastically explore it; if it didn't work, it didn't work. But often enough, some idea that seemed far-fetched did work. We both had the freedom to continue without worrying about where it came from or where it would lead. It was exciting to work with Sir Michael all these years. And it is as true today as it was when we first met in '55 - that sense of excitement and "anything goes" and "let's see what happens".



ATIYAH [...] there are a lot of people who contributed in the background to the build-up of the Index Theorem – going back to Abel, Riemann, much more recently Serre, who got the Abel prize last year, Hirzebruch, Grothendieck and Bott. There was lots of work from the algebraic geometry side and from topology that prepared the ground. And of course there are also a lot of people who did fundamental work in analysis and the study of differential equations: Hörmander, Nirenberg... In my lecture I will give a long list of names[2]; even that one will be partial. It is an example of international collaboration; you do not work in isolation, neither in terms of time nor in terms of space – especially in these days. Mathematicians are linked so much, people travel around much more. We two met at the Institute at Princeton. It was nice to go to the Arbeitstagung in Bonn every year, which Hirzebruch organised and where many of these other people came. I did not realize that at the time, but looking back, I am very surprised how quickly these ideas moved...

Collaboration seems to play a bigger role in mathematics than earlier. There are a lot of conferences, we see more papers that are written by two, three or even more authors – is that a necessary and commendable development or has it drawbacks as well?

ATIYAH It is not like in physics or chemistry whereyou have 15 authors because they

[1]More details were given in the laureates' lectures.

[2]Among those: Newton, Gauss, Cauchy, Laplace, Abel, Jacobi, Riemann, Weierstrass, Lie, Picard, Poincaré, Castelnuovo, Enriques, Severi, Hilbert, Lefschetz, Hodge, Todd, Leray, Cartan, Serre, Kodaira, Spencer, Dirac, Pontrjagin, Chern, Weil, Borel, Hirzebruch, Bott, Eilenberg, Grothendieck, Hörmander, Nirenberg

need an enormous big machine. It is not absolutely necessary or fundamental. But particularly if you are dealing with areas which have rather mixed and interdisciplinary backgrounds, with people who have different expertise, it is much easier and faster. It is also much more interesting for the participants. To be a mathematician on your own in your office can be a little bit dull, so interaction is stimulating, both psychologically and mathematically. It has to be admitted that there are times when you go solitary in your office, but not all the time! It can also be a social activity with lots of interaction. You need a good mix of both, you can't be talking all the time. But talking some of the time is very stimulating. Summing up, I think that it is a good development – I do not see any drawbacks.

SINGER Certainly computers have made collaboration much easier. Many mathematicians collaborate by computer instantly; it's as if they were talking to each other. I am unable to do that. A sobering counterexample to this whole trend is Perelman's results on the Poincaré conjecture: He worked alone for ten to twelve years, I think, before putting his preprints on the net.

ATIYAH Fortunately, there are many different kinds of mathematicians, they work on different subjects, they have different approaches and different personalities – and that is a good thing. We do not want all mathematicians to be isomorphic, we want variety: different mountains need different kinds of techniques to climb.

SINGER I support that. Flexibility is absolutely essential in our society of mathematicians. [. . .]

Mathematics and physics

We would like to have your comments on the interplay between physics and mathematics. There is Galilei's famous dictum from the beginning of the scientific revolution, which says that the Laws of Nature are written in the language of mathematics. Why is it that the objects of mathematical creation, satisfying the criteria of beauty and simplicity, are precisely the ones that time and time again are found to be essential for a correct description of the external world? Examples abound, let me just mention group theory and, yes, your Index Theorem!

SINGER There are several approaches in answer to your questions; I will discuss two. First, some parts of mathematics were created in order to describe the world around us. Calculus began by explaining the motion of planets and other moving objects. Calculus, differential equations, and integral equations are a natural part of physics because they were developed for physics. Other parts of mathematics are also natural for physics. I remember lecturing in Feynman's seminar, trying to explain anomalies. His postdocs kept wanting to pick coordinates in order to compute; he stopped them saying:

Fotograf: *Abelprisen/DNVA*

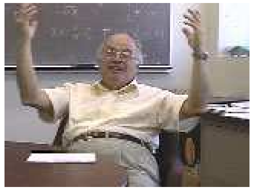
"The Laws of Physics are independent of a coordinate system. Listen to what Singer has to say, because he is describing the situation without coordinates." Coordinate-free means geometry. It is natural that geometry appears in physics, whose laws are independent of a coordinate system.

Symmetries are useful in physics for much the same reason they're useful in mathematics. Beauty aside, symmetries simplify equations, in physics and in mathematics. So physics and math have in common geometry and group theory, creating a close connection between parts of both subjects.

Secondly, there is a deeper reason if your question is interpreted as in the title of Eugene Wigner's essay "The Unreasonable Effectiveness of Mathematics in the Natural Sciences[3]". Mathematics studies coherent systems which I will not try to define. But it studies coherent systems, the connections between such systems and the structure of such systems. We should not be too surprised that mathematics has coherent systems applicable to physics. It remains to be seen whether there is an already developed coherent system in mathematics that will describe the structure of string theory. [At present, we do not even know what the symmetry group of string field theory is.] Witten has said that 21st century mathematics has to develop new mathematics, perhaps in conjunction with physics intuition, to describe the structure of string theory.

ATIYAH I agree with Singer's description of mathematics having evolved out of the physical world; it therefore is not a big surprise that it has a feedback into it. More fundamentally: to understand the outside world as a human being is an attempt to reduce complexity to simplicity. What is a theory? A lot of things are happening in the outside world, and the aim of scientific inquiry is to reduce this to as simple a number of principles as possible.

[3]Comm. Pure App. Math. 13(1), 1960 31/08/04 12



That is the way the human mind works, the way the human mind wants to see the answer.

If we were computers, which could tabulate vast amounts of all sorts of information, we would never develop theory – we would say, just press the button to get the answer. We want to reduce this complexity to a form that the human mind can understand, to a few simple principles. That's the nature of scientific inquiry, and mathematics is a part of that. Mathematics is an evolution from the human brain, which is responding to outside influences, creating the machinery with which it then attacks the outside world. It is our way of trying to reduce complexity into simplicity, beauty and elegance. It is really very fundamental, simplicity is in the nature of scientific inquiry – we do not look for complicated things.

I tend to think that science and mathematics are ways the human mind looks and experiences – you cannot divorce the human mind from it. Mathematics is part of the human mind. The question whether there is a reality independent of the human mind, has no meaning – at least, we cannot answer it. [. . .]

Newer developments

Can we move to newer developments with impact from the Atiyah-Singer Index Theorem? I.e., String Theory and Edward Witten on the one hand and on the other hand Non-commutative Geometry represented by Alain Connes. Could you describe the approaches to mathematical physics epitomized by these two protagonists?

ATIYAH I tried once in a talk to describe the different approaches to progress in physics like different religions. You have prophets, you have followers – each prophet and his followers think that they have the sole possession of the truth. If you take the strict point of view that there are several different religions, and that the intersection of all these theories is empty, then they are all talking nonsense. Or you can take the view of the mystic, who thinks that they are all talking of different aspects of reality, and so all of them are correct. I tend to take the second point of view. The main "orthodox" view among physicists is certainly represented by a very large group of people working with string theory like Edward Witten. There are a small number of people who have different philosophies, one of them is Alain Connes, and the other is Roger Penrose. Each of them has a very specific point of view; each of them has very interesting ideas. Within

the last few years, there has been non-trivial interaction between all of these.

They may all represent different aspects of reality and eventually, when we understand it all, we may say "Ah, yes, they are all part of the truth". I think that that will happen. It is difficult to say which will be dominant, when we finally understand the picture – we don't know. But I tend to be open-minded. The problem with a lot of physicists is that they have a tendency to "follow the leader": as soon as a new idea comes up, ten people write ten or more papers on it and the effect is that everything can move very fast in a technical direction. But big progress may come from a different direction; you do need people who are exploring different avenues. And it is very good that we have people like Connes and Penrose with their own independent line from different origins. I am in favour of diversity. I prefer not to close the door or to say "they are just talking nonsense".

SINGER String Theory is in a very special situation at the present time. Physicists have found new solutions on their landscape - so many that you cannot expect to make predictions from String Theory. Its original promise has not been fulfilled. Nevertheless, I am an enthusiastic supporter of Super String Theory, not just because of what it has done in mathematics, but also because as a coherent whole, it is a marvellous subject. Every few years new developments in the theory give additional insight. When that happens, you realize how little one understood about String Theory previously. The theory of D-branes is a recent example. Often there is mathematics closely associated with these new insights. Through D-branes, K-theory entered String Theory naturally and reshaped it. We just have to wait and see what will happen. I am quite confident that physics will come up with some new ideas in String Theory that will give us greater insight into the structure of the subject, and along with that will come new uses of mathematics.

Alain Connes' program is very natural – if you want to combine geometry with quantum mechanics, then you really want to quantize geometry, and that is what non-commutative geometry means. Non-commutative Geometry has been used effectively in various parts of String Theory explaining what happens at certain singularities, for example. I think it may be an interesting way of trying to describe black holes and to explain the Big Bang. I would encourage young physicists to understand non-commutative geometry more deeply than they presently do. Physicists use only parts of non-commutative geometry; the theory has much more to offer. I do not know whether it is going to lead anywhere or not. But one of my projects is to try and redo some known results using noncommutative geometry more fully. [...]

Communication of mathematics

Next topic: Communication of mathematics: Hilbert, in his famous speech at the International Congress in 1900, in order to make a point about mathematical communication, cited a French mathematician who said: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street". In order to pass on to new generations of mathematicians the collective knowledge of the previous generation, how important is it that the results have simple and elegant proofs?

ATIYAH The passing of mathematics on to subsequent generations is essential for the future, and this is only possible if every generation of mathematicians understands what they are doing and distils it out in such a form that it is easily understood by the next generation. Many complicated things get simple when you have the right point of view. The first proof of something may be very complicated, but when you understand it well, you readdress it, and eventually you can present it in a way that makes it look much more understandable – and that's the way you pass it on to the next generation! Without that, we could never make progress - we would have all this messy stuff. Mathematics does depend on a sufficiently good grasp, on understanding of the fundamentals so that we can pass it on in as simple a way as possible to our successors. That has been done remarkably successfully for centuries. Otherwise, how could we possibly be where we are? In the 19th century, people said: "There is so much mathematics, how could anyone make any progress?" Well, we have - we do it by various devices, we generalize, we put all things together, we unify by new ideas, we simplify lots of the constructions – we are very successful in mathematics and have been so for several hundred years. There is no evidence that this has stopped: in every new generation, there are mathematicians who make enormous progress. How do they learn it all? It must be because we have been successful communicating it.

SINGER I find it disconcerting speaking to some of my young colleagues, because they have absorbed, reorganized, and simplified a great deal of known material into a new language, much of which I don't understand. Often I'll finally say, "Oh; is that all you meant?" Their new conceptual framework allows them to encompass succinctly considerably more than I can express with mine. Though impressed with the progress, I must confess impatience because it takes me so long to understand what is really being said.

Has the time passed when deep and important theorems in mathematics can be given short proofs? In the past, there are many such examples, e.g., Abel's one-page proof of the addition theorem of algebraic differentials or Goursat's proof of Cauchy's integral theorem.

ATIYAH I do not think that at all! Of course, that depends on what foundations you are allowed to start from. If we have to start from the axioms of mathematics, then every proof will be very long. The common framework at any given time is constantly advancing; we are already at a high platform. If we are allowed to start within that framework, then at every stage there are short proofs.

One example from my own life is this famous problem about vector fields on spheres solved by Frank Adams where the proof took many hundreds of pages. One day I discovered how to write a proof on a postcard. I sent it over to Frank Adams and we wrote a little paper which then would fit on a bigger postcard. But of course that used some K-theory; not that complicated in itself. You are always building on a higher platform; you have always got more tools at your disposal that are part of the lingua franca which you can use. In the old days you had a smaller base: If you make a simple proof nowadays, then you are allowed to assume that people know what group theory is, you are allowed to talk about Hilbert space. Hilbert space took a long time to develop, so we have got a much bigger vocabulary, and with that we can write more poetry.

SINGER Often enough one can distil the ideas in a complicated proof and make that part of a new language. The new proof becomes simpler and more illuminating. For clarity and logic, parts of the original proof have been set aside and discussed separately.

ATIYAH Take your example of Abel's Paris memoir: His contemporaries did not find it at all easy.

It laid the foundation of the theory. Only later on, in the light of that theory, we can all say: “Ah, what a beautifully simple proof!” At the time, all the ideas had to be developed, and they were hidden, and most people could not read that paper. It was very, very far from appearing easy for his contemporaries.

Individual work style [. . .]

SINGER I seem to have some built-in sense of how things should be in mathematics. At a lecture, or reading a paper, or during a discussion, I frequently think, “that’s not the way it is supposed to be.” But when I try out my ideas, I’m wrong 99% of the time. I learn from that and from studying the ideas, techniques, and procedures of successful methods. My stubbornness wastes lots of time and energy. But on the rare occasion when my internal sense of mathematics is right, I’ve done something different.

Both of you have passed ordinary retirement age several years ago. But you are still very active mathematicians, and you have even chosen retirement or visiting positions remote from your original work places. What are the driving forces for keeping up your work? Is it wrong that mathematics is a “young man’s game” as Hardy put it?

ATIYAH It is no doubt true that mathematics is a young man’s game in the sense that you peak in your twenties or thirties in terms of intellectual concentration and in originality. But later you compensate that by experience and other factors. It is also true that if you haven’t done anything significant by the time you are forty, you will not do so suddenly. But it is wrong that you have to decline, you can carry on, and if you manage to diversify in different fields this gives you a broad coverage. The kind of mathematician who has difficulty maintaining the momentum all his life is a person who decides to work in a very narrow field with great depths, who e.g. spends all his life trying to solve the Poincaré conjecture – whether you succeed or not, after 10-15 years in this field you exhaust your mind; and then, it may be too late to diversify. If you are the sort of person that chooses to make restrictions to yourself, to specialize in a field, you will find it harder and harder – because the only things that are left are harder and harder technical problems in your own area, and then the younger people are better than you. [. . .]

Apart from mathematics. . .

Could you tell us in a few words about your main interests besides mathematics?

SINGER I love to play tennis, and I try to do so 2-3 times a week. That

refreshes me and I think that it has helped me work hard in mathematics all these years.

ATIYAH Well, I do not have his energy! I like to walk in the hills, the Scottish hills – I have retired partly to Scotland. In Cambridge, where I was before, the highest hill was about this (gesture) big. Of course you have got even bigger ones in Norway. I spent a lot of my time outdoors and I like to plant trees, I like nature. I believe that if you do mathematics, you need a good relaxation which is not intellectual – being outside in the open air, climbing a mountain, working in your garden. But you actually do mathematics meanwhile. While you go for a long walk in the hills or you work in your garden – the ideas can still carry on. My wife complains, because when I walk she knows I am thinking of mathematics.

SINGER I can assure you, tennis does not allow that!

Thank you very much on behalf of the Norwegian, the Danish, and the European Mathematical Societies!

Arrangementer

Complex Days of the North - Journees Complexes du Nord
Reykjavik, Iceland, January 4-5, 2005

I forbindelse med at de franske og islandske matematiske foreningene arrangerer den 24. nordiske og den første fransk-nordiske matematikkkongress vil det være en “pre-congress satellite meeting on complex analysis”.

Participants can leave continental Europe or North America on Monday January 3 (there is a direct flight from Paris to Reykjavik on that day, by the way) and enjoy two days of complex analysis before the main congress.

Look up the travel and satellite meeting information on the website of the congress (note the special offer for participants flying from Paris). E-mail me at `ragnar@hi.is` *if you are interested in participating or giving a lecture.*

For generell informasjon om kongressen, se `http://www.raunvis.hi.is/1FrancoNor`